A NOTE ON THE BEHAVIOR OF AN IDEALIZED HALF-WAVE MAGNETIC AMPLIFIER IN THE PRESENCE OF EDDY CURRENTS

BY
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ABSTRACT

The steady state behavior of a half-wave magnetic amplifier, under certain idealized assumptions for the B-H relationship and the eddy current effect, is studied in order to provide a model which may serve as a guide to the understanding of the more complex phenomena which occur in actual circuits.

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Introduction and Assumptions

The steady state behavior of a half-wave magnetic amplifier, under certain idealized conditions, is studied in order to provide a model which may serve as a guide to the understanding of the more complex phenomena which occur in actual circuits. The B-H relationship is represented by a single-valued step function having zero and infinite slopes in the saturated and unsaturated regions respectively; the eddy current loss is represented by a constant resistance connected across an additional winding which links the main core flux.

The half-wave magnetic amplifier considered is shown in Fig. MRI-13342-a; mesh currents, voltages, and resistances are expressed on a per unit turn basis. The following assumptions are made in the analysis:

1. Leakage fluxes are neglected throughout.

2. The B-H relationship* under cyclic operation is represented by a single-valued, step-shaped curve displaced to the left of the ordinate axis by an amount equal to the "coercive" force of the material. (The equivalent Φ versus i_m relationship, shown in Fig. MRI-13342-b, is used in this paper as a matter of convenience.) Geometric effects are neglected.

3. The rectifier has infinite resistance to the flow of reverse current and a constant resistance to the flow of forward current. Rectifier forward resistance and winding resistance of the load coil are lumped together with the load resistance.

4. The control circuit current is smooth d-c (the control mesh is completely constrained).

5. The minimum cyclic core flux, Φ_0, is always greater than $(-\Phi_s)$.

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* It should be made very clear that in the term, "B-H relationship", the "H" here refers to the magnetizing force produced by the total ampere turns acting on the core, including eddy current ampere turns. A plot of flux density versus total applied (accessible to measurement) ampere turns, such as results from the well known a-c oscillograph method, is here referred to as an "apparent" B-H loop (or B-H relationship). The B-H relationship for a magnetic material can be measured with accuracy only in the d-c case. The a-c or cyclic B-H relationship cannot be obtained by direct measurement because the component of magnetizing force due to eddy currents is inaccessible; hence, all a-c measurements lead to "apparent" B-H relationships which are necessarily dependent upon the test circuitry and conditions of excitation. The a-c B-H relationship could be obtained from the apparent B-H relationship only if appropriate corrections could be made for the influence of eddy currents in the core.

6. The eddy current loss is simulated by a fixed resistance* connected across an additional winding which links the main core flux.

Analysis

From Fig. MRI-13342-a, the basic equations are written as follows:

$$v_m \sin \omega t = ri + \frac{d\Phi}{dt} \quad \text{for } i \geq 0, \tag{1}$$

$$0 = r_e i_e + \frac{d\Phi}{dt}, \tag{2}$$

$$i_m = i + i_e + I_c. \tag{3}$$

Three distinct modes of operation must be considered; these are:

I. Core saturated;

$$t_1 \leq t \leq t_2 \quad \text{and} \quad \Phi = \Phi_s, \quad \frac{d\Phi}{dt} = 0, \quad i_m \geq I_K,$$

$$i \geq 0, \; i_e = 0.$$

From Eq. (1),

$$i = \frac{v_m}{r} \cdot \sin \omega t. \tag{4}$$

II. Core not saturated and $i \neq 0$;

$$t_2 \leq t \leq t_3 \text{ and } t_4 \leq t \leq (t_1 + T), \text{ and}$$

$$\Phi \leq \Phi_s, \; i_m = I_K.$$

From Eq. (2),

$$\frac{d\Phi}{dt} = -r_e i_e, \tag{5a}$$

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* This simplification cannot be justified on a theoretical basis. The equivalent eddy current resistance is assumed to be a constant for a specific lamination thickness, magnetic material, interlaminar insulation, and excitation frequency.

and from Eq. (3),

$$i + i_e = (I_K - I_C) = D, \text{ by definition.} \tag{6a}$$

Over the region of useful amplifier control,

$$D \geq 0.$$

From Eqs. (1), (2), (5a), and (6a), one obtains by simple algebra,

$$\frac{d\Phi}{dt} = \frac{r_e}{r + r_e} (v_m \cdot \sin \omega t - r\,D) \tag{5}$$

and

$$i = \frac{1}{r + r_e} \cdot (v_m \cdot \sin \omega t + r_e D). \tag{6}$$

III. Core not saturated and $i = 0$;

$$t_3 \leq t \leq t_4 \quad \text{and} \quad \Phi \leq \Phi_s, \quad i_m = I_K, \quad i_e = D.$$

Then,

$$\frac{d\Phi}{dt} = -r_e D. \tag{7}$$

TABLE I

Summary of Important Formulae

Region	I	II	III
i	$\frac{v_m}{r} \cdot \sin \omega t$	$\frac{1}{r+r_e}(v_m \sin \omega t + r_e D)$	0
i_e	0	$\frac{-1}{r+r_e}(v_m \sin \omega t - r\,D)$	D
$\frac{d\Phi}{dt}$	0	$\frac{r_e}{r+r_e}(v_m \sin \omega t - r\,D)$	$-r_e D$
t	t_1 to t_2	t_2 to t_3, t_4 to (t_1+T)	t_3 to t_4.

Transition from mode I to mode II occurs when $i = D$ at $t = t_2$. Thus, from Table I,

$$\sin \theta_2 = \frac{r\,D}{V_m} . \tag{8}$$

(In general, $\theta = \omega t$.)

Similarly, the transition from mode II to mode III occurs when $i = 0$ at $t = t_3$; therefore,

$$\sin \theta_3 = -\frac{r_e D}{V_m} ; \tag{9}$$

or,

$$\frac{\sin \theta_2}{\sin \theta_3} = -\frac{r}{r_e} . \tag{10}$$

It is seen by inspection of Table I that

$$\sin \theta_4 = \sin \theta_3 ; \tag{11}$$

hence,

$$\theta_4 = 3\pi - \theta_3 . \tag{11a}$$

If the core flux attains its minimum cyclic value at $t = t_5$, $\frac{d\Phi}{dt} = 0$, and from the table,

$$\sin \theta_5 = \sin \theta_2 , \tag{12}$$

and,

$$\theta_5 = 3\pi - \theta_2 . \tag{12a}$$

Waves of i and i_e, and Φ are shown in Figs. MRI-13343-a and MRI-13343-b, respectively.

Two restrictions must be imposed upon the circuit parameters; these are:

a) $\frac{r\,D}{V_m} \leq 1$ and b) $\frac{r_e D}{V_m} \leq 1$.

If condition (a) is violated, the core never saturates. If condition (b) is violated, the core reaches the negative saturation branch of the B-H curve.

The saturation angle θ_1 (corresponding to t_1) can be related to known quantities through the equation expressing the periodicity of flux,

$$\int_{t_2}^{t_3} \frac{d\Phi}{dt}\,dt + \int_{t_3}^{t_4} \frac{d\Phi}{dt}\,dt + \int_{t_4}^{t_1+T} \frac{d\Phi}{dt}\,dt = 0 \;; \tag{13}$$

after some algebra, this leads to the relationship,

$$v_m \cos\theta_1 + rD\,(\theta_1-\theta_2) = v_m(\cos\theta_2 - 2\cos\theta_3) + r_e D(2\,\theta_3 - 3\pi). \tag{14}$$

Average Load Current:

The rectified average value of load current, defined by

$$\bar{I} = \frac{1}{T}\left(\int_{t_1}^{t_2} i.dt + \int_{t_2}^{t_3} i.dt + \int_{t_4}^{t_1} i.dt\right) \quad ,$$

is found to be

$$\frac{\bar{I}}{\bar{I}_s} = -\cos\theta_3 - \sin\theta_3\,.\,(\theta_3 - \frac{3}{2}\pi)\,. \tag{15}$$

A more convenient form is obtained by introducing

$$\delta = \theta_3 - \pi\,.$$

Hence,

$$\frac{\bar{I}}{\bar{I}_s} = \cos\delta + \sin\delta\,.\,(\delta - \frac{\pi}{2})\;, \tag{15a}$$

where,

$$\delta = \sin^{-1}\frac{r_e D}{v_m}\,. \tag{16}$$

For small values of δ,

$$\frac{\bar{I}}{\bar{I}_s} \doteq 1 - \frac{\pi}{2}\,\delta + \frac{\delta^2}{2}\,, \tag{15b}$$

and

$$\delta \doteq \frac{r_e D}{v_m}\,. \tag{16a}$$

Minimum Core Flux:

The minimum core flux Φ_o at time t_5 is obtained by integrating

$$\Phi_o - \Phi_s = \int_{t_2}^{t_3} \frac{d\Phi}{dt}\cdot dt + \int_{t_3}^{t_4} \frac{d\Phi}{dt}\cdot dt + \int_{t_4}^{t_5} \frac{d\Phi}{dt}\cdot dt\,. \tag{17}$$

This leads to the result

$$(\Phi_o - \Phi_s) = \frac{-1}{\omega}\cdot\frac{r_e}{r_e + r}\cdot\left[2v_m(\cos\theta_3 - \cos\theta_2) - D(r(2\theta_2 - 3\pi) + r_e(2\theta_3 - 3\pi))\right]. \tag{18}$$

A more compressed expression is obtained by introducing:

$$\theta_3 = \pi + \delta\,, \quad \theta_2 = \pi - \gamma\,, \quad \frac{r}{r_e} = C, \quad \text{and}$$

$$K = \frac{\Phi_m}{\Phi_s} = \frac{v_m}{\omega\Phi_s}\,;$$

$$\sin\gamma = C\sin\delta\,.$$

Eq. (18) becomes

$$\frac{\Phi_o}{\Phi_s} = 1 + \frac{2K}{1+C}\cdot\left[(\cos\delta - \cos\gamma) + ((\delta - \tfrac{\pi}{2}) - C(\gamma + \tfrac{\pi}{2}))\cdot\sin\delta\right]\,. \tag{18a}$$

In the usual case $C \ll 1$ and the above relationship is further simplified to the form,

$$\frac{\Phi_o}{\Phi_s} \doteq 1 + 2K \left[(\cos\delta - 1) + (\delta - \frac{\pi}{2}) \cdot \sin\delta \right] . \qquad (18b)$$

A convenient parameter to be used in plotting the results is the term,

$$K \sin\delta = \frac{r_e D}{\omega\Phi_s} .$$

Solutions for the critical values of flux, Φ_3 and Φ_4, are given in Appendix A.

Remarks

If the eddy current effects were neglected, the transfer curve of the ideal magnetic amplifier would have infinite slope over the useful control range; with eddy currents present, the transfer curve (i.e., $\bar{I}$ vs. I_c) possesses finite slope over the entire control range. A "Master" Transfer Curve which plots $\bar{I}/\bar{I}_s$ versus sin δ (i.e., $r_e D/v_m$) is shown in Fig. MRI-13344. The quantity "D" may now be considered as the horizontal shift of a point on the transfer curve (for a specified value of $\bar{I}$) caused by eddy currents. The shift D approaches zero as $R_e \rightarrow \infty$.

It has been recognized[1,2] for some time that the relationship between minimum flux and control current (Control Magnetization Curve or CMC) is one possible means for describing the a-c behavior of the near-rectangular-loop magnetic materials. As the excitation frequency approaches zero, the CMC approaches the descending branch of a large d-c loop. A "Master" CMC which plots Φ_o/Φ_s versus $r_e D/\omega\Phi_s$, for the case c $\doteq$ 0 (practical case), is shown in Fig. MRI-13345. The Master CMC is seen to be a function of K (or v_m); however, when $K > 1.5$ the change in the CMC with further increase in K or a-c voltage is small. Here "D" may be interpreted as the horizontal displacement (at a specified value of Φ_o) between the CMC and the descending branch of the true B-H loop.

It is interesting to note that although a single-valued B-H relationship was assumed in the analysis, the "apparent" B-H relationship, obtained by plotting Φ versus $(i + i_c)$, is a loop as shown in Fig. MRI-13346 (shown for $I_K = 0$). Therefore, any attempt to evaluate magnetic characteristics through the measurement of a-c loops must be made with great caution.

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1 H. Lehmann, AIEE Trans., 1951, 70, p. 2097.

2 R. Zarouni, Research Report R-288-52, PIB-227, O.N.R. Contract N6ori-98, Task Order IV.

APPENDIX A

$$\frac{\Phi_s - \Phi_3}{\Phi_s} = \frac{K}{1 + C} \cdot \left[(\cos \theta_3 - \cos \theta_2) - C(\theta_3 - \theta_2) \cdot \sin \theta_3\right] \tag{A1}$$

$$= \frac{K}{1 + C} \cdot \left[(\cos \gamma - \cos \delta) + C(\gamma + \delta) \cdot \sin \delta\right], \tag{A1a}$$

and for $C \ll 1$,

$$\frac{\Phi_s - \Phi_3}{\Phi_s} \doteq K\,(1 - \cos \delta). \tag{A1b}$$

$$\frac{\Phi_3 - \Phi_4}{\Phi_s} = - K\,(3\pi - 2\theta_3) \cdot \sin \theta_3. \tag{A2}$$

$$= K\,(\pi - 2\delta) \cdot \sin \delta\ ; \tag{A2a}$$

for $\delta \ll 1$,

$$\frac{\Phi_3 - \Phi_4}{\Phi_s} \doteq \pi k \cdot \sin \delta, \tag{A2b}$$

$$\Phi_3 - \Phi_4 \doteq \frac{r_e D}{2f}\ . \tag{A2c}$$

$$\frac{\Phi_4 - \Phi_o}{\Phi_s} = \frac{K}{1 + C} \cdot \left[(\cos \theta_3 - \cos \theta_2) + (\theta_3 - \theta_2) \cdot \sin \theta_2\right] \tag{A3}$$

$$= \frac{K}{1 + C} \cdot \left[(\cos \gamma - \cos \delta) + C\,(\gamma + \delta) \sin \delta\right]\ ; \tag{A3a}$$

for $C << 1$,

$$\frac{\Phi_4 - \Phi_0}{\Phi_s} \doteq K\,(1 - \cos\delta). \tag{A3b}$$

Therefore,

$$(\Phi_s - \Phi_3) = (\Phi_4 - \Phi_0). \tag{A4}$$

In Eq. (18b) when K is large

$$\frac{\Phi_s - \Phi_0}{\Phi_s} \doteq \pi K \sin\delta = \frac{\pi r_e D}{\omega\,\Phi_s}, \tag{A5}$$

and

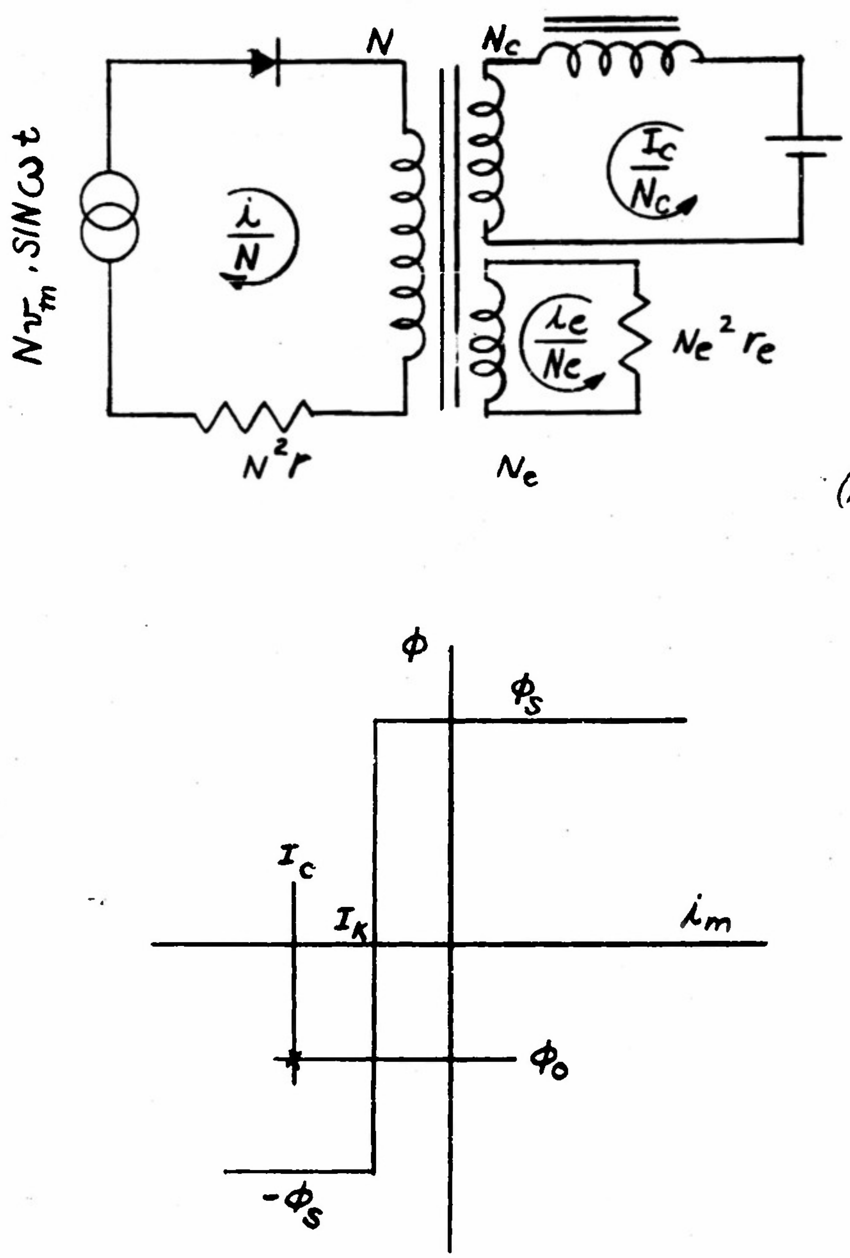
$$\Phi_s - \Phi_0 = \frac{r_e D}{2f}. \tag{A5a}$$

Load current corresponding to Φ_0 occurs when $t = t_5$; from Table I,

$$i_0 = \frac{1}{r + r_e}\,(v_m \sin\theta_5 + r_e\,D)$$

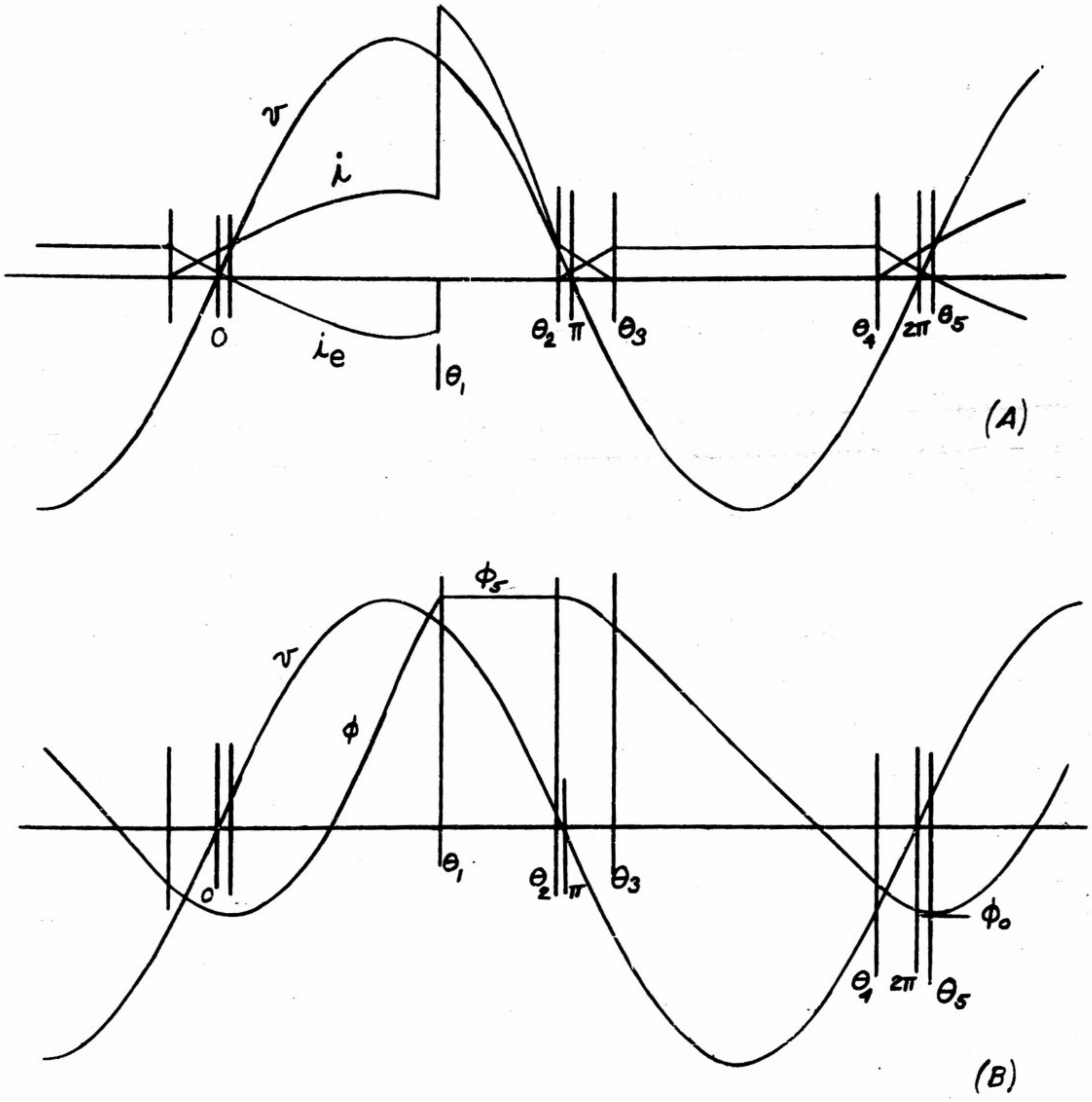
which reduces to

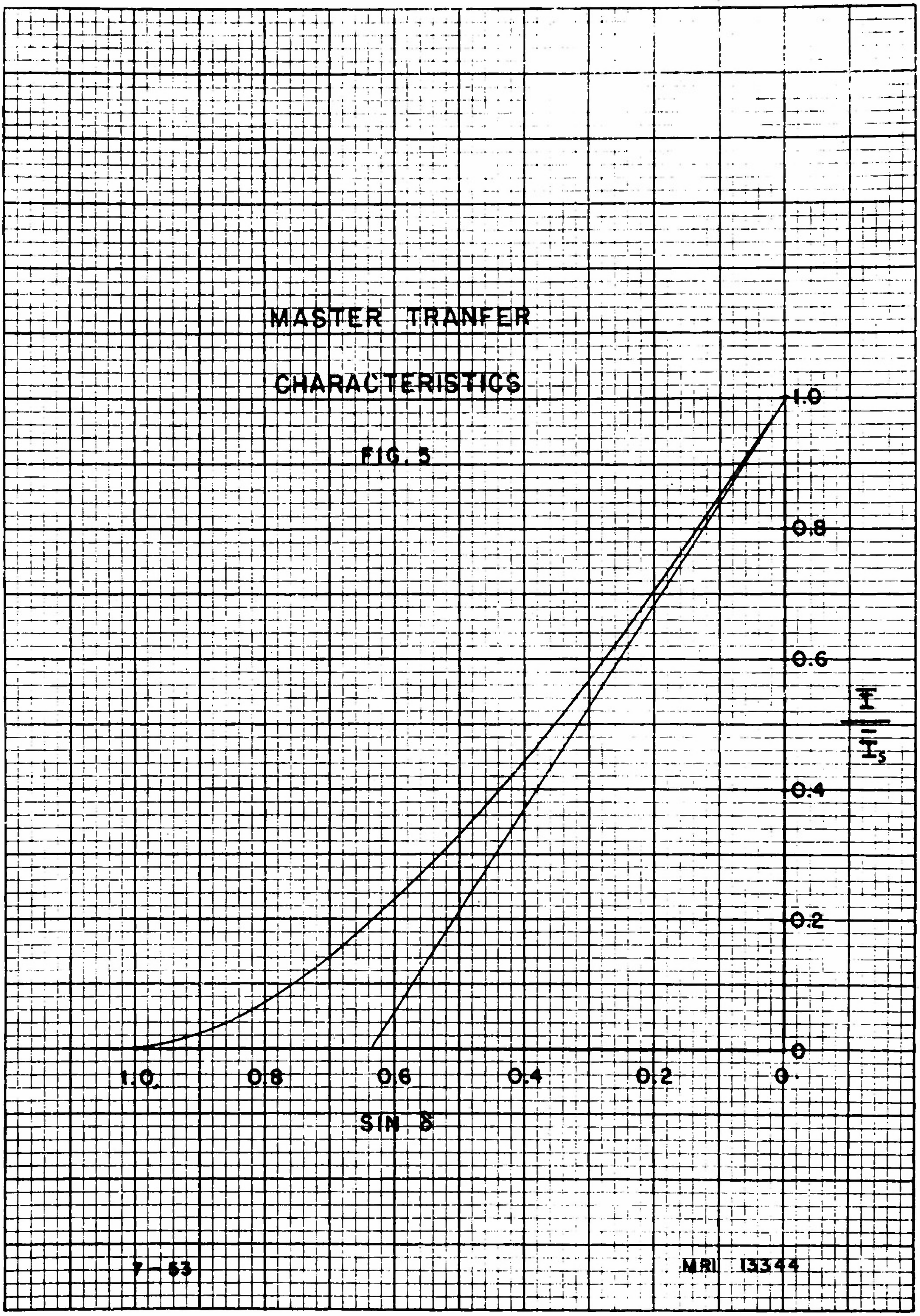
$$i_0 = D. \tag{A6}$$


N
N_c
$\frac{I_c}{N_c}$
$\frac{i}{N}$
$N v_m \sin \omega t$
$\frac{i_e}{N_e}$
$N_e^2 r_e$
$N^2 r$
N_e
ϕ
ϕ_s
I_c
I_K
i_m
ϕ_o
$-\phi_s$

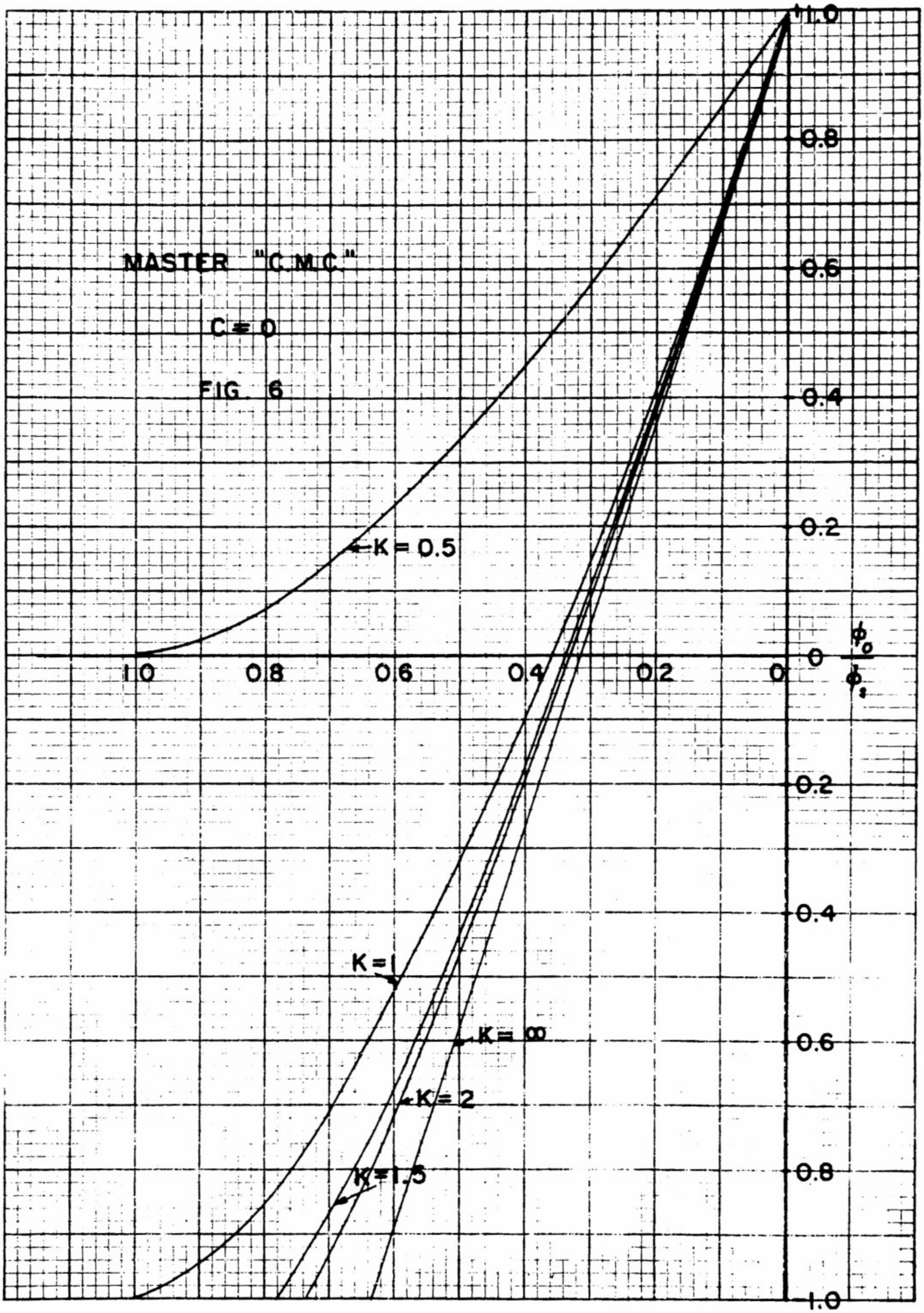
(A)

(B)

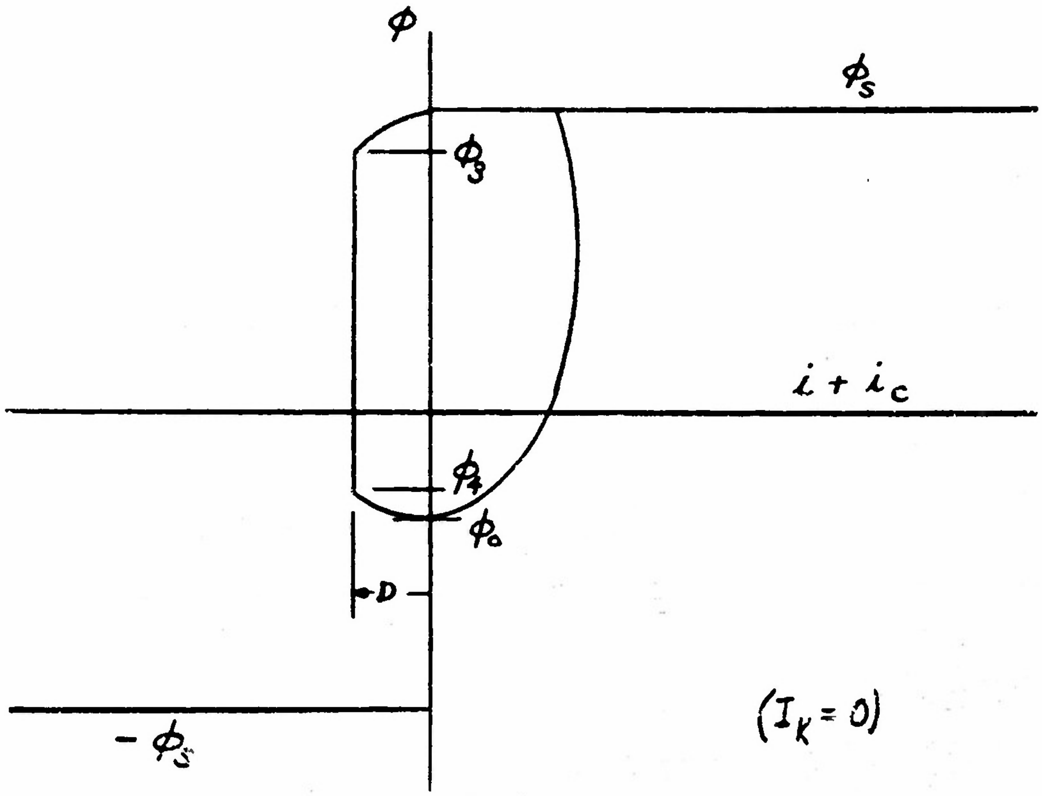

v
i
i_e
0
θ_1
θ_2
π
θ_3
θ_4
2π
θ_5
(A)
ϕ_s
v
φ
0
θ_1
θ_2
π
θ_3
ϕ_o
θ_4
2π
θ_5
(B)



MASTER TRANFER
CHARACTERISTICS
FIG. 5
1.0
0.8
0.6
0.4
0.2
0
$\frac{I}{I_s}$
1.0
0.8
0.6
0.4
0.2
0
SIN δ
7-53
MRI 13344



MASTER "C.M.C."
C = 0
FIG. 6
K = 0.5
K = 1
K = ∞
K = 2
K = 1.5
1.0
0.8
0.6
0.4
0.2
0
$\frac{\phi_o}{\phi_s}$



APPARENT B-H LOOP

FIG. 7